

$$15) \begin{aligned} g(a) &= -3a^2 - a \\ h(a) &= -2a - 4 \end{aligned}$$

Find $\left(\frac{g}{h}\right)(a)$

$$\frac{-3a^2 - a}{-2a - 4}$$

$$17) \begin{aligned} h(a) &= 3a \\ g(a) &= -a^3 - 3 \end{aligned}$$

Find $\left(\frac{h}{g}\right)(a)$

$$\frac{3a}{-a^3 - 3}$$

$$19) \begin{aligned} h(x) &= x^2 - 2 \\ g(x) &= 4x + 1 \end{aligned}$$

Find $(h \circ g)(x)$

$$16x^2 + 8x - 1$$

$$16) \begin{aligned} f(n) &= 2n \\ g(n) &= -n - 4 \end{aligned}$$

Find $(f \circ g)(n)$

$$-2n - 8$$

$$18) \begin{aligned} g(n) &= 2n + 3 \\ h(n) &= n - 1 \end{aligned}$$

Find $(g \circ h)(n)$

$$2n + 1$$

$$20) \begin{aligned} g(t) &= 2t + 5 \\ f(t) &= -t^2 + 5 \end{aligned}$$

Find $(g + f)(t)$

$$-t^2 + 2t + 10$$

16) $f(n) = 2n$

$g(n) = -n - 4$

Find $(f \circ g)(n)$

$z(-n-4)$

$f(g(n))$

$$15) \quad g(a) = -3a^2 - a$$

$$h(a) = -2a - 4$$

$$\text{Find } \left(\frac{g}{h}\right)(a) =$$

$$\frac{-3a^2 - a}{-2a - 4}$$

$$\frac{-3a^2 - a}{-2a - 4}$$

$$\frac{g(a)}{h(a)} = \frac{-3a^2 - a}{-2a - 4} = \frac{-a(3a + 1)}{-2(a + 2)}$$

$$21) \begin{aligned} g(x) &= 2x - 2 \\ f(x) &= x^2 + 3x \\ \text{Find } (g \circ f)(-2 + x) \\ &2x^2 - 2x - 6 \end{aligned}$$

$$23) \begin{aligned} g(x) &= 2x + 3 \\ f(x) &= 3x^2 - 3x \\ \text{Find } -4g(-4x) + 4f(-4x) \\ &192x^2 + 80x - 12 \end{aligned}$$

$$25) \begin{aligned} g(x) &= x^3 + 3 \\ h(x) &= 3x + 2 \\ \text{Find } (3g + 3h)(-x) \\ &-3x^3 - 6x + 13 \end{aligned}$$

$$27) \begin{aligned} g(t) &= t^3 - 3t^2 \\ f(t) &= -t - 4 \\ \text{Find } g(-2t) - f(-2t) \\ &-8t^3 - 12t^2 - 2t + 4 \end{aligned}$$

$$22) \begin{aligned} g(a) &= 2a + 2 \\ h(a) &= -2a - 5 \\ \text{Find } (g \circ h)(-4 + a) \\ &-4a + 8 \end{aligned}$$

$$24) \begin{aligned} g(t) &= 3t - 1 \\ f(t) &= 3t^3 + t \\ \text{Find } (3g + 3f)(4t) \\ &192t^3 + 40t - 3 \end{aligned}$$

$$26) \begin{aligned} f(t) &= t - 4 \\ g(t) &= t^3 - 3 \\ \text{Find } (f \cdot g)(-2 - t) \\ &t^4 + 12t^3 + 48t^2 + 83t + 66 \end{aligned}$$

$$28) \begin{aligned} f(x) &= 2x + 2 \\ g(x) &= -3x - 1 \\ \text{Find } \left(\frac{f}{g}\right)(-4x) \\ &\frac{-8x + 2}{12x - 1} \end{aligned}$$

24) $g(t) = 3t - 1$

$f(t) = 3t^3 + t$

Find $(3g + 3f)(4t)$

$192t^3 + 40t - 3$

$3(3(4t) - 1) + 3(3(4t)^3 + (4t))$

$3(3t + 1) + 3(3t^3 + 1)$

$9t + 3 + 9t^3 + 3$

$9t^3 + 9t + 6$

$9(4t)^3 + 9(4t) + 6$

$9(4^3 t^3) + 36t + 6$

$9(64t^3) + 36t + 6$

$576t^3 + 36t + 6$

29) $f(x) = -x - 4$
 $g(x) = 2x^2 - 2$
Find $f(-2x) - g(-2x)$
 $-8x^2 + 2x - 2$

30) $g(n) = n^2 - 5n$
 $h(n) = 2n + 1$
Find $g(y - 2) - h(y - 2)$
 $y^2 - 11y + 17$

COMPOSITION OF FUNCTIONS Functions can also be combined using **composition of functions**. In a composition, a function is performed, and then a second function is performed on the result of the first function. The composition of f and g is denoted by $f \circ g$.

Key Concept

Composition of Functions

Suppose f and g are functions such that the range of g is a subset of the domain of f . Then the composite function $f \circ g$ can be described by the equation

$$[f \circ g](x) = f[g(x)].$$

Composite Functions

Date _____

Perform the indicated operation.

1) $f(a) = a^2 - 3a$
 $g(a) = a + 3$
Find $(f \circ g)(-10)$

70

3) $g(n) = 2n - 4$
 $h(n) = 3n + 3$
Find $(g \circ h)(7)$

44

5) $h(n) = 4n$
 $g(n) = -3n - 5$
Find $h(g(-8))$

76

7) $f(x) = x - 4$
 $g(x) = x^3 + 5x$
Find $f(g(3))$

38

9) $f(n) = -n - 1$
 $g(n) = -4n + 3$
Find $f(g(n - 4))$

$4n - 20$

2) $g(t) = t + 3$
 $h(t) = t^2 - 4t$
Find $(g \circ h)(0)$

3

4) $h(n) = 3n - 1$
 $g(n) = 3n + 4$
Find $h(g(-4))$

-25

6) $g(n) = 4n - 5$
 $h(n) = n^2 - n$
Find $(g \circ h)(-4)$

75

8) $f(n) = -2n - 1$
Find $(f \circ f)(-7)$

-27

10) $h(n) = n^2 - n$
 $g(n) = n - 2$
Find $h(g(-2n))$

$4n^2 + 10n + 6$

8) $f(n) = -2n - 1$
Find $(f \circ f)(-7)$
-27

$$\begin{aligned} & f(f(-7)) \\ & -2(-2(-7) - 1) - 1 \\ & -2(14 - 1) - 1 \\ & -2(13) - 1 \\ & -26 - 1 = -27 \end{aligned}$$

-23

6) $g(n) = 4n - 5$
 $h(n) = n^2 - n$
Find $(g \circ h)(-4)$

75

6) $g(n) = 4n - 5$ $g(h(-4))$
 $h(n) = n^2 - n$ $(-4)^2 - (-4) = 12$
Find $(g \circ h)(-4)$ $4(20) - 5 = \text{~~75~~}$

$80 - 5 = 75$

$(-4)^2 - (-4)$
 $16 + 4$
 20

3) $g(n) = 2n - 4$
 $h(n) = 3n + 3$
Find $(g \circ h)(7)$

44

$$h(g(x)) = (h \circ g)(x)$$

$$3(2(7) - 4) + 3$$

$$14 - 4$$

$$3(10) + 3$$

$$33$$

$$g(h(7))$$

$$3(7) + 3$$

$$21 + 3$$

$$2(24) - 4$$

$$48 - 4$$

$$44$$

2) $g(t) = t + 3$
 $h(t) = t^2 - 4t$
Find $(g \circ h)(0)$

$$g(h(0))$$
$$(0^2 - 4(0)) + 3$$
$$0 + 3$$
$$3$$

1) $f(a) = a^2 - 3a$
 $g(a) = a + 3$
Find $(f \circ g)(-10)$
70
 $f(g(-10))$

$$g \circ f = g(f(-10))$$

$$f(-10 + 3)$$

$$f(-7)$$

$$(-7)^2 - 3(-7)$$

$$49 + 21$$

$$70$$

9) $f(n) = -n - 1$
 $g(n) = -4n + 3$
 Find $f(g(n - 4))$
 $4n - 20$

11) $h(a) = -4a - 2$
 $g(a) = a^3 - 5a^2$
 Find $h(g(-1 - a))$
 $4a^3 + 32a^2 + 52a + 22$

13) $g(n) = 4n + 4$
 $f(n) = n^2 + 3n$
 Find $(g \circ f)(-n)$
 $4n^2 - 12n + 4$

15) $g(x) = x^2 - 2x$
 $f(x) = 3x + 4$
 Find $g(f(-2t))$
 $36t^2 - 36t + 8$

10) $h(n) = n^2 - n$
 $g(n) = n - 2$
 Find $h(g(-2n))$
 $4n^2 + 10n + 6$

12) $g(x) = -4x - 3$
 Find $g(g(-x))$
 $-16x + 9$

14) $g(x) = -x^2 + 4$
 $f(x) = x - 4$
 Find $(g \circ f)(4x)$
 $-16x^2 + 32x - 12$

16) $h(n) = n^2 - 4$
 $g(n) = -4n - 5$
 Find $h\left(g\left(\frac{n}{3}\right)\right)$
 $\frac{189 + 120n + 16n^2}{9}$